

LDPC Performance Over the 802.11n Protocol

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Abstract— Low Density Parity Check Codes (LDPC) is a set of algorithms that send, receive and correct in a noise environment, frames transmitted in a LAN environment. This article demonstrates the high performance of the LDPC in environments of noise, compared to the CRC error detection code highly currently implemented, in this way the efficiency of LDPC is shown specifically over the 802.11n protocol. (Abstract)

Keywords—LDPC, Shanon limit, 802.11 and signals.

I. INTRODUCTION

In 1984, Shannon demonstrates that there is a limit to the amount of information that can be transmitted reliably without errors, with presence of Gaussian noise in the environment, without stating a specific algorithm that manages to reach this limit. This limit gave way to create specific algorithms with runtimes of polynomial order, which could reach, or be very close to its maximum capacity of transmission of information with little probability of errors, so in 1963 Robert G. Gallager first created the Low Density Parity-Check Codes that comply with this, hereafter treated as LDPC for its acronym in English.

Due to the advancement of the technology at the time, both the LPDC and the Shannon limit could not be implemented until some years later when they had access to wireless communication devices. In principle this article will show the importance of these algorithms illustrating their great contribution to the transmission of data, for this, first the Shannon limit will be discussed, and later on the different algorithms used in LDPC will be defined, just as the performance will be analyzed and finally its simulation and analysis.

II. LIMIT OF SHANNON

Claude Shannon established a mathematical theory that says: "the limit of the effective data rate of a channel depends on the wide bandwidth and signal to noise ratio" where a given channel with noise with capacity C, is defined with the following formula:

$$C = W \log_2 \left(1 + \frac{P}{N} \right) \quad (1)$$

Where:

- W is the width of the channel.
- C is the capacity of the channel.
- P is the useful signal strength.
- N is the power of noise present in the channel.

According to Shannon, if there is a channel of communication with a capacity C and a binary rate R, where:

$$R < C \quad (2)$$

R: Binary rate, Bits per second

Then, there is a technique of coding that allows the probability of the error to be arbitrarily small. This theorem shows that there is a technique, without stating any specifically. On the basis of this Gallager found a group of algorithms that is approaching the Shannon limit in a very efficient manner, these are the LDPC algorithms [1] [12].

III. PERFORMANCE OF SIGNALS WITH NOISE

To measure the performance, it is necessary to calculate the BER (Bit Error Rate) which identifies the number of incorrect bits over the number of bits sent in a transmission with noise.

And calculating the ratio of power noise, which is defined in the following manner:

$$\square \frac{E_b}{N_0} \quad (3)$$

Where:

E_b = The average energy per bit signal

N_0 = It is the variance of the noise

This is related to the Shannon limit when the data (R) transmission rate is small (approaching zero) compared to the bandwidth, you will reach the next limit, occasionally called ultimate Shannon limit: [15]

$$\frac{E_b}{N_0} > \ln(2) \approx -1,59 \text{ dB} \quad (4)$$

In other words, the equation above says that while using an infinitely large bandwidth, reliable transmission is achieved if the $\frac{E_b}{N_0}$ is approximately -1.59 DB.

On the contrary, if R is limited, it is found that the Shannon limit in terms is high, for example if R=1/2 the limit becomes $\ln \frac{E_b}{N_0} > 1$ (0 dB).

Depending on the used modulation and the $\frac{E_b}{N_0}$ a probability with the limit of Shannon's reliable information delivery can be found. [11]

IV. DEFINITION OF LDPC

They are linear codes that allow the transmission of messages through a noisy channel whose essential property is to have at least one array of low parity, it means with "few" elements different from zero. [17] This matrix ensures that its complexity grows linearly with the length of the code, and at a minimum distance that also grows according to the length of the code [3].

Generally it is used the following notation to represent them, LDPC (n, j, k) showing a code of block length n, and (j, k) represent the number of rows and columns respectively [1]

V. CONSTRUCTION OF LDPC

There are two ways for the construction of low parity matrix:

- Random building codes
- LDPC codes of structured Construction

Even though random building codes may have better performance than the structured construction, these last are most appropriate in terms of constructive encoder complexity.

The construction proposed by D. Mackay for example is random in nature, while other approaches such as Shu Lin's [14] are based on geometric considerations, as well as others in the conformation of the matrix with cyclic or quasi-cyclic properties. There is also another approach within the LDPC codes of structured construction, based on Combinatory mathematic (known as balanced incomplete block design); for the construction of LDPC. [13]

This article will work in this case with the Mackay and Neal method, since it is one of the most widely used because of its better performance.

a) tered on the page; all previous will be in two columns.

A. MacKay and Neal method

In this method the columns of the matrix of low parity that we will call H are added one by one from left to right. The weight of each column is chosen to obtain the degree of distribution and the position of the different from 0 entries, in each column chosen at random among those rows that have not

completed their degree yet. If at any time there are rows with more vacancies, then columns must be added, you can repeat the process if some columns do not have the desired degree. For this type of matrices it is not regular, the only constant degree is that of columns, the pseudocode can be found in the references [3].

ENCODING

The Generating matrix can be found with Gauss-Jordan elimination to get the form:

$$H = [A, I_{n-k}] \quad (5)$$

Where is A a binary matrix (n-k)*k.

The Generating matrix is:

$$G = [I_k, A^T] \quad (6)$$

Given a matrix G and a p message, the message encoded in module 2 is given by:

$$c = Gp \quad (7)$$

Given a received y message, it is defined:

$$z = Hy \quad (8)$$

If z=0, it means that the message has been received error free. [7]

VI. DECODING

There are different ways of decoding, but they are divided into two large parts, "Hard-decision" and "soft-decision", in this document only the first is covered.:

A. Tanner Graph

It is represented by a matrix where the rows represent the check nodes and columns are the bits nodes, in this way we can obtain from the matrix:

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Fig. 1: LDPC matrix

The following graph of Tanner:

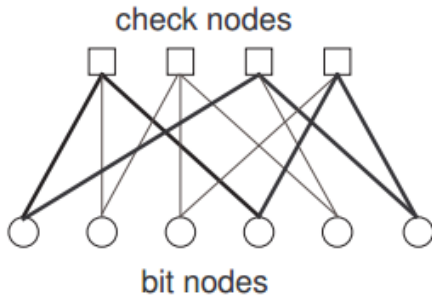


Fig. 2: Representation of matrix of low parity [3]

B. Hard-Decision

With a H matrix we get that the message received had a distortion in a bit, the algorithm that must be followed is as follows:

All nodes bit sent a message to its nodes checks. In that case the message, is the bit that node believed correct to that node check.

Each check node calculates its reply of the message according to the other messages received, this response can be either 0 or 1.

All bit nodes are forced to connect with a check node in particular through the calculation of the equation "parity-check" through the partial sum with module 2, and it saves the value of the sum in a cell of memory associated with its respective check. Then the edges through which messages were sent are deleted.

It Spreads towards the variables the value of the partial sum of the checks which has degree 1 (i.e. receiving the value of all variables involved in the equation subtracting one, therefore the missing one is determined). The value of the variables that receive values through any of their edges is determined by the received value and the edges through which values were sent are deleted.

If, in step 3, the message was sent, return to step 2.

The exit is called \hat{x}^n , and returns the associated values to each variable (some can be determined and other deleted).

Since each edge is used as Maximum once it has a complexity of linear algorithm.

VII. SIMULATION

A simulation in MATLAB R2013b was done, in the part of creation, they are restricted to a matrix of LDPC with different rates of code that uses the IEEE standard protocol 802. 11n [9].

The code rate is represented in the following way:

$$R = \frac{1}{n} \quad (9)$$

Which means that per each original bit it is needed to encode on n additional bits.

For the generation of G, it is used the method of factorization LU [8].

Since the standard does not specify the decoding type used, it is supposed to use the so-called Hard-decision method, more specifically bit-flit, in addition it is performed according to BPSK, QPSK, and 64-QAM modulation. [9]

FER vs $\frac{E_b}{N_0}$ with a dotted line, and the BER vs $\frac{E_b}{N_0}$, is shown with a continuous line. You can see in Figure 3. (BER) that the bit error rate is in a range of 10^{-1} in a plot with 648 bit block length (this length is given by the IEEE standard, according to the rate of the code). [9]

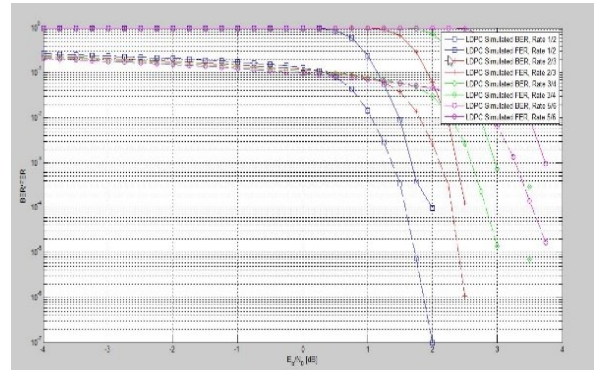


Fig. 3: Simulation of LDPC 802. 11n, with different encoding rates.

Having different encoding rates, it was found that 2 dB are needed to achieve a quality of transfer with probability of 10^{-7} . Although the maximum quality for larger ratios as it is $\frac{5}{6} y^{\frac{3}{4}}$ is of 10^5 , with a transmission power which increases nearly double, but that definitely is a very small power compared with the increase of the number of bits sent.

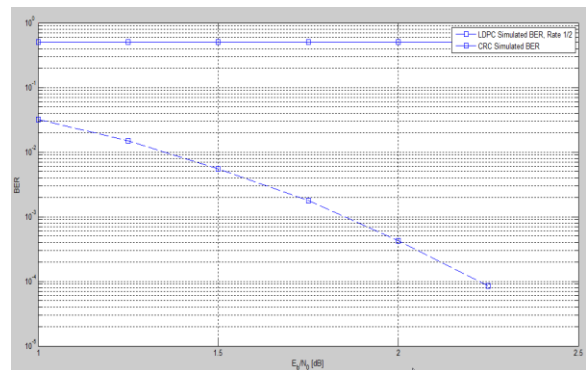


Fig. 4: Efficiency LDPC vs CRC - 50 iterations.

In addition a comparison with the CRC was made, which is generalized for 802.3 frames, in this way you can see in Figure 4. The efficiency of this algorithm in a noise environment. This comparison was carried out with a rate of 1/2, which according to the above is the most effective, but you can easily see that

their BER average result is 0,4999 in comparison with the results of the LDPC with a BER of 0.0091 on average. This proves to us that its efficiency is really low, and even increasing the signal strength and noise this will hardly decrease.

CONCLUSIONS

The LDPC efficiency was shown with different rates of coding in a 802. 11n protocol, simulated in Matlab, regarding this it is shown in the following table Summary:

TABLE I. COMPARISON OF EFFICIENCY OF LDPC, WITH DIFFERENT ENCODING RATES

	BER~.	dB
Rate ½	10⁻⁷	2
Rate 2/3	10⁻⁶	2,5
Rate ¾	10⁻⁵	3
Rate 5/6	10⁻⁵	3,7

In this way it shows that at a rate of 1/2 coding we have a BER of 10^{-7} for example. In addition compared to the CRC the LDPC has an advantage in the order of e^{-4} , implying that in LAN data networks it is much more efficient and presents a better performance when transmitting, using a correction code as the LDPC.

Future Works: In future works, they must simulate the different methods of modulation for each of the rates, and identify the optimal BER of each one.

REFERENCES

[1] Robert G. Gallager, Low-Density Parity-Check , (1963) Available at : <http://www.rle.mit.edu/rgallager/documents/ldpc.pdf>

[2] Alix Lhéritier , Inst. Ingenieria de computacion, Univ. de la Republica, Uruguay . Available at: <http://www.fing.edu.uy/~alherit/informe.pdf>

[3] Sarah J. Johnson , Introducion Low-Density Parity-Check Codes , Available at: <http://sigpromu.org/sarah/SJohnsonLDPCintro.pdf>

[4] IEEE P802.11 Wireless LANs, "TGn Channel Models", IEEE 802.11-03/940r4, 2004-05-10.

[5] Claude E. Shannon , Communication in the Presence of Noise.(1998) Available at: <http://www.stanford.edu/class/ee104/shannonpaper.pdf>

[6] Marcos martin Fernandez, Señales aleatorias y ruido, Univ. de Valladolid.

[7] Tuan Ta, A tutorial on Low Density Parity-Check Codes Available at: <http://www.ece.umd.edu/~tta/resources/LDPC.pdf>

[8] J.L Vieitez, Descomposición LU de matrices (2006). Available at: http://www.fing.edu.uy/imerl/metnum/archivos/material_teorico/LU-vieitez.pdf

[9] IEEE Standard for information technology – Telecommunications and information exchange between systems – Local and metropolitan area networks – Specific requirements – Part 11. (2009)

[10] G. Cherubini, N. Benvenuto Algorithms for communications Systems and their applications, (2002).

[11] David J.C MacKay, Rdford M. Neal .Near Shannon Limit Performance of Low Density Parity Check Codes , Available at: <http://www.cs.toronto.edu/~mackay/mncEL.pdf>

[12] L. J. Arnone, Transmisión segura en comunicaciones inalámbricas de corto alcance. (2008). Available at: http://www3.fi.mdp.edu.ar/electronica/tesisgradoyposgrado/tesis/Tesis_Arnone_Leonardo.pdf

[13] Yu Kou , Shu Lin. Low density parity check codes based on finite geometries a rediscovery and New Results. Available at: http://www.stanford.edu/class/ee379b/class_reader/ucd1.pdf

[14] Cap 4. The gap between uncoded performance and the Shannon Limit, Available at: <http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-451-principles-of-digital-communication-ii-spring-2005/lecture-notes/chap4.pdf>

[15] Artés , F. Pérez, Comunicaciones digitales (2012), Available at: http://www.tsc.uc3m.es/~antonio/libro_comunicaciones/El_libro_files/c_omdig_artes_perez.pdf

[16] Bernhard M.J Leiner ,LDPC Codes (2005). Available at: <http://www.bernh.net/media/download/papers/ldpc.pdf>