Determination of game-based design equilibria by using Machine Learning approach

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***Abstract*— This paper presents a comprehensive study on the application of artificial intelligence and machine learning to enhance efficiency and precision in game-based design problems, with specific focus on pressure vessels, bilevel problems with three followers, and speed reducers. It proposes an AI-enhanced machine learning framework to solve numerically complex optimization engineering designs problems beyond traditional methods. This approach is demonstrated through three example problems, each viewed as a game with set players, presenting unique challenges and design requirements. The process begins by developing datasets from specific problem intervals and features, using MATLAB tools to achieve optimized solutions. These optimized results then serve as training data for a neural network, designed to predict rational reaction sets of players involved in the design process, thereby facilitating more informed and accurate decision-making. By integrating advanced machine learning techniques and formulating problems through game theory, this approach not only streamlines the computational process but also significantly improves the reliability and adaptability of engineering solutions. This research introduces the transformative impact of machine learning in game-based design, offering adaptive, efficient, and robust design optimizations that provides a new era in the field.**

Keywords— ***artificial intelligence, game theory, machine learning, design decision-making***

I. INTRODUCTION

Optimization in engineering design[1] is crucial for enhancing efficiency and precision across various industries, including manufacturing[2], systems engineering[3], aerospace[4], automotive[5], and civil engineering[6]. These fields demand rigorous computational methods to meet high standards of performance and reliability. Traditional optimization approaches often face significant challenges due to their resource-intensive and time-consuming nature[7]. As a result, there is a continuous search for innovative methods that can improve the effectiveness and efficiency of the optimization process. Game theory, a mathematical framework for analyzing strategic interactions among rational decision-makers, where there are multiple objective functions in the problem, has been increasingly applied to engineering design problems[8], [9], [10], [11]. By modeling engineering challenges as games, where each component or stakeholder can be considered a player with specific objectives and constraints, game theory provides a structured approach to finding optimal solutions that account for the interdependencies and competitive nature of real-world systems[12], [13]. This perspective allows for more comprehensive and balanced optimization, ensuring that the interests of all parties are considered and harmonized.

The advent of artificial intelligence (AI) has brought a transformative impact on numerous fields, including engineering design[14], [15], [16], [17]. AI offers advanced analytical and predictive capabilities that can handle complex and high-dimensional data, making it an invaluable tool for solving intricate design problems. By automating and enhancing various aspects of the design process, AI can significantly reduce the time and resources required to achieve optimal solutions while improving the overall accuracy and robustness of the results.

Machine learning (ML), a subset of AI, further enhances the optimization process by enabling systems to learn from data and improve their performance over time without explicit programming[18]. ML techniques, such as neural networks, decision trees, and reinforcement learning, can identify patterns and relationships within large datasets, providing insights that are often beyond the reach of traditional methods. When applied to engineering design, ML can optimize complex systems by predicting outcomes and suggesting improvements based on historical data and simulations.

This paper introduces a novel AI-enhanced machine learning framework aimed at optimizing the gamed based design problem. To show how this method is working, the approach has been applied in three specific engineering problems: pressure vessel optimization, bilevel problems with three followers[19], and speed reducer design[20]. By modeling these problems as games with defined players and utilizing MATLAB tools to generate detailed datasets, it derives optimized solutions used to train a neural network. This network predicts rational reaction sets of the players, enhancing decision-making accuracy and efficiency. Incorporating game theory and advanced machine learning techniques, this approach reduces computational overhead and improves the reliability of engineering designs, demonstrating significant improvements over traditional methods. This research highlights the transformative impact of AI and ML in game-based design optimization, marking a significant advancement in predictive optimization for engineering design.

In Section II, it discusses the proposed methodology, detailing the innovative approach developed for addressing game theory problems in engineering design. Section III presents a series of examples and the corresponding results obtained through the methodology. Finally, in Section IV, it discusses the implications of our findings and potential avenues for future research.

II. METHOD

A. *Game Theory and Rational Reaction Sets*

Game theory is a mathematical framework that analyzes strategic interactions among rational decision-makers, known as players. Each player in a game aims to maximize their payoff while considering the actions and reactions of other players. The concept of a rational reaction set (RRS) refers to the set of strategies that rational players would choose in response to the strategies of other players. In the context of engineering design problems, each component or stakeholder can be modeled as a player with specific objectives and constraints, and the design problem can be structured as a game. There are three types of games: cooperative game, non-cooperative (Nash) game, and an extensive game.

Nash Equilibrium: In a non-cooperative game, each player has a set of variables under its control and optimizes its objective function individually. The player does not care how its selection affects the payoff functions of other players. The players bargain with each other to obtain an equilibrium solution, if one exists. This solution is called Nash solution. the use of Nash solutions in solving design problems was proposed by Vincent [21]. In 1987, the concept of the Nash solution was expanded by Rao to encompass games involving multiple players, extending beyond its original scope of two-player scenarios[22]. Further examples of employing game theory in mechanical design can be found in[23], [24].

Stackelberg equilibrium: Stackelberg games are an important type of extensive-form games[25]. In leader-follower games, also known as Stackelberg games, one player (the leader) makes a decision first, and the other players (followers) make their decisions subsequently, knowing the leader's decision. This hierarchical decision-making process reflects many real-world scenarios where certain decisions are made sequentially rather than simultaneously. The leader's goal is to anticipate the followers' responses and optimize their own decision accordingly. The followers then optimize their strategies based on the leader's decision. The Stackelberg equilibrium is the solution to this type of game, where the leader and the followers reach a stable state where no player can improve their payoff by unilaterally changing their strategy, given the strategies of the other players. This equilibrium considers the leader's advantage of moving first and the followers' rational reactions.

The interactions among the RRS are crucial for understanding the dynamic and interdependent nature of the design process. Traditional approaches to solving these interactions include:

Design of experience: The design of experience approach for game theory involves structuring experiments to explore various strategies and outcomes in strategic interactions among decision-makers[26]. This approach focuses on creating controlled environments where participants engage in decision-making processes akin to those encountered in real-world scenarios. Through these experiments, researchers gain insights into the dynamics of strategic behavior, equilibrium solutions, and the impact of different factors on decision-making.

Sensitivity-Based: These methods focus on understanding how changes in design variables impact the overall system performance. By analyzing the sensitivity of the system to various parameters, designers can identify critical variables and optimize them effectively[19]. However, these methods can be limited by the complexity and non-linearity of interactions among variables.

Optimization Algorithms: Techniques such as genetic algorithms[27], simulated annealing[28], and gradient-based methods have been employed to find approximate solutions to game-based problems. These methods can be computationally expensive and may not always converge to the optimal solutions.

Simulation-Based Methods: Monte Carlo simulations and other probabilistic methods are used to explore the strategy space and estimate optimal strategies[29]. These methods require significant computational resources and may suffer from scalability issues.

B. *AI-Enhanced Machine Learning Framework*

This approach leverages artificial intelligence (AI) and machine learning (ML) to find rational reaction sets (RRS) in complex engineering design problems. The proposed framework consists of the following steps as shown in Fig. 1.

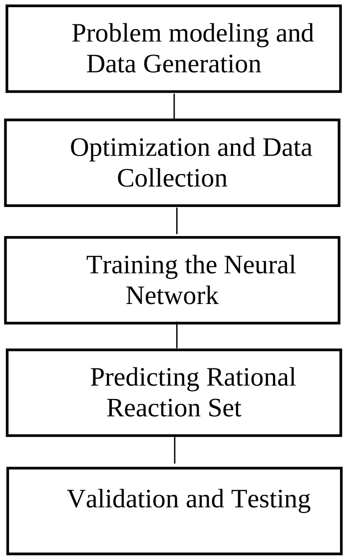

Fig. 1. Proposed framework for engineering design optimization using ML.

Problem Modeling and Data Generation: Each engineering problem is modeled as a game with defined players. MATLAB tools have been utilized to develop datasets that capture the specific problem intervals, features, and constraints. These datasets represent various scenarios and outcomes of the game.

Design Variables: For each problem, specific design variables are identified. These variables are critical parameters that influence the performance and outcomes of the design.

Optimization and Data Collection: Using MATLAB optimization tools, it derives optimized solutions for the given engineering problems. These solutions serve as the initial dataset, capturing the interactions and strategies of the players under different conditions.

Training the Neural Network: The optimized dataset is used to train a neural network. The neural network learns to predict the rational reaction sets of the players based on the input features. This training process involves:

Data Preprocessing: Normalizing and scaling the data to ensure efficient training.

Model Selection: Choosing an appropriate neural network architecture, such as feedforward neural networks or recurrent neural networks, depending on the complexity of the problem.

Training and Validation: Splitting the dataset into training and validation sets to assess the model's performance and prevent overfitting.

Predicting Rational Reaction Sets: Once trained, the neural network predicts the rational reaction sets for new, unseen problem instances. This prediction helps in understanding the strategic interactions among the players and informs better decision-making.

Validation and Testing: The predicted RRS are validated against additional test cases and compared with traditional optimization methods to evaluate the efficiency and accuracy of our approach. Performance metrics such as prediction accuracy, computational time, and robustness are analyzed.

III. EXAMPLES AND RESULTS

To validate the AI-enhanced machine learning framework, This approach has been tested on three specific engineering problems: pressure vessel optimization, bilevel problems with three followers, and speed reducer design. Below, it starts by providing a detailed description of the pressure vessel optimization problem as an illustrative example of how our framework can be applied.

A. Pressure Vessel Optimization

The optimization of pressure vessels is a classic problem in multi objective engineering design problems, involving the determination of the optimal design parameters to minimize the cost while meeting safety and performance requirements. The design of pressure vessels typically involves variables such as wall thickness, internal pressure, material properties, and vessel dimensions[19]. In this example, it demonstrates how the AI-enhanced machine learning framework can be applied to optimize the design of a pressure vessel, considering it as a game with multiple players.

Problem Formulation: Consider the problem of designing a thin-walled pressure vessel with three design variables: the radius R, the length L, and the thickness T. This problem has been used as a test problem in literature by several researchers[24], [30]. The two objective functions include maximizing the volume (VOL) and minimizing the weight (WGT) of the vessel. Player 1 (VOL) wishes to maximize the volume by controlling variables R and L, whereas Player 2 (WGT) minimizes the weight with control over variable T. The vessel is under internal pressure P.

Constraints: The problem constraints are given in Eq. (1) to Eq. (4).

$$\sigma_{CIRC} = \frac{PR}{T} \leq S \quad (1)$$

$$5 - T - R \leq 0 \quad (2)$$

$$R + T - 40 \leq 0 \quad (3)$$

$$L + 2R + T - 150 \leq 0 \quad (4)$$

Objective Functions: The mathematical form of the problems for players VOL and WGT are given in Eq. (5) and Eq. (6):

$$V(R, L) = \pi R^2 L \quad (5)$$

$$W(R, T, L) = 2\pi RTL + 2\pi R^2 T \quad (6)$$

Optimization: To explore the dynamics of leader-follower interactions in the game-based design optimization process, it is conducted two separate analyses, each treating one of the objective functions (VOL and WGT) as the leader while the other function served as the follower. First, it is considered Player 1 (VOL) as the leader and Player 2 (WGT) as the follower. It is formulated the optimization problem with the objective of maximizing the volume (VOL) of the pressure vessel while satisfying the given constraints using MATLAB optimization tools. By varying the design variables R and L, it is derived the optimal solution for Player 1. Subsequently, it is reversed the roles, with Player 2 (WGT) acting as the leader and Player 1 (VOL) as the follower. In this scenario, the objective was to minimize the weight (WGT) of the pressure vessel by adjusting the design variable T, subject to the same constraints as before. Again, it is utilized MATLAB optimization tools to solve the optimization problem for Player 2.

Data Generation: Using MATLAB optimization tools, it is derived optimized solutions for each leader-follower scenario. These solutions capture the interactions and strategies of the players (design variables) under different conditions. The optimized results serve as the initial dataset for training the neural network.

Training the Neural Network: For the neural network part, it is adopted a feedforward approach. After generating the initial dataset from the optimized solutions obtained, it is proceeded to train the neural network. This training process involved preprocessing the data by normalizing and scaling it to ensure efficient training. Next, an appropriate neural network architecture has been selected, opting for a feedforward neural network given its suitability for the problem's complexity.

Data Integration: The RRS obtained from the neural network represent the strategic responses of the follower to the leader's decisions. These RRS are integrated into the leader's optimization problem. With the RRS incorporated, the leader's optimization problem is solved again. This time, the leader considers the strategic responses of the follower when making decisions to achieve the best outcome for its objective function.

The AI-enhanced method yielded optimal parameters of R=36.25 inches, L=69.75 inches and T=4.1inches as are shown in Table I. Comparatively, the AI-enhanced method demonstrated fewer iterations for convergence and showcased greater robustness to parameter variations. While effective, the sensitivity-based approach required more iterations for convergence and exhibited sensitivity to numerical perturbations in regression coefficients.

TABLE I

Sensitivity-based approach	**AI-Enhanced approach**
R=35.99inch	R=36.25inch
L=70inch	L=69.75inch
T=4inch	T=4.1inch

B. Bilevel Problem with Three Followers

Consider a bilevel problem with one leader and three followers where the followers have non-cooperative game among themselves. The leader has control over variables $x = (x_1, x_2)$ and aims to optimize a composite objective function given in Eq. (7) with the constraints given in Eq. (8) and Eq. (9). The first follower controls variables $y_1 = (y_{11}, y_{12})$ with the objective function given in Eq. (10) its constraints are outlined in Eq. (11), Eq. (12) and Eq. (13). The second follower controls variables $y_2 = (y_{21}, y_{22})$ with the objective function given in Eq. (14) and its constraint is outlined in Eq. (15). The third follower controls variables $y_3 = (y_{31}, y_{32})$ with the objective function given in Eq. (16) its constraints are outlined in Eq. (17) and Eq. (18).

$$f_1(x, y_1, y_2, y_3) = \frac{3(y_{11} + y_{12})^2 + 5(y_{21} + y_{22})^2 + 10(y_{31} + y_{32})^2}{2{x_1}^2 + {x_2}^2 + 3x_1x_2} \quad (7)$$

Subject to $$x_1 + 2x_2 \leq 10 \quad (8)$$

$$x_1, x_2 > 0 \quad (9)$$

$$f_1(y_1) = {y_{11}}^2 + {y_{12}}^2 \quad (10)$$

Subject to $$y_{11} + y_{21} + y_{31} \geq x_1 \quad (11)$$

$$y_{12} + y_{22} + y_{32} \geq x_2 \quad (12)$$

$$y_{11} \geq 1, \quad y_{12} \geq 2 \quad (13)$$

$$f_2(y_2) = y_{21} + y_{22} + \frac{y_{11}}{y_{21}} + \frac{y_{12}}{y_{22}} \quad (14)$$

Subject to $$y_{11}, y_{22} > 0 \quad (15)$$

$$f_3(y_3) = \frac{(y_{31} - y_{21})^2}{y_{31}} + \frac{(y_{32} - y_{22})^2}{y_{32}} \quad (16)$$

Subject to $$2y_{31} + 3y_{32} = 5 \quad (17)$$

$$y_{31}, y_{32} > 0 \quad (18)$$

Initially, each follower independently optimizes its objective function using MATLAB optimization tools, resulting in optimal solutions ${y_1}^*, {y_2}^*$ and ${y_3}^*$. The optimal solutions obtained from the followers are utilized to generate a comprehensive dataset, capturing various scenarios and outcomes of the optimization process. A neural network is trained using this dataset to predict the rational reaction sets (RRS) of the followers in response to the leader's decisions. This involves preprocessing the data, selecting an appropriate neural network architecture, training and validation phases. The RRS obtained from the neural network are then incorporated into the leader's optimization problem. This allows the leader to anticipate and account for the followers' responses in its decision-making process. Using the proposed AI-enhanced approach, the following results have been obtained and shown in Table II.

TABLE II

	Sensitivity-based Approach	**AI-enhanced Approach**
f_1	1.510	1.501
f_1	10821	12.323
f_2	6.061	6.225
f_3	0.483	0.834
$x^* = ({x_1}^*, {x_2}^*)$	(5.379,2.310)	(5.768,2.116)
${y_1}^* = ({y_{11}}^*, {y_{12}}^*)$	(2.612,2.00)	(2.885,2.116)
${y_2}^* = ({y_{21}}^*, {y_{22}}^*)$	(1.616,1.414)	(1.699,1.414)
${y_1}^* = ({y_{31}}^*, {y_{32}}^*)$	(1.149,0.900)	(1.183,0.789)

The results obtained from our AI-enhanced approach closely align with those from the sensitivity-based approach presented by [19]. This suggests that the method effectively captures the dynamics of the bilevel optimization problem.

C. Speed Reducer Optimization Problem

Consider a two-objective optimization problem for a speed reducer system, comprising a leader and two subsystem followers. The leader controls variables $x = (x_1, x_2, x_3, x_4, x_5, x_6, x_7)$ and aims to minimize the total volume of the speed reducer as well as the maximum stress in the first or second gear shaft. The leader's objective function is given in Eq. (19).

$$f_1 = 0.7854x_1{x_2}^2\left(\frac{10{x_3}^2}{3} + 14.933x_3 - 43.0934\right) - 1.508x_1({x_6}^2 + {x_7}^2) + 7.477({x_6}^3 + {x_7}^3) + 0.7854(x_4{x_6}^2 + x_5{x_7}^2) \quad (19)$$

$$f_2 = \max\{f_{12}, f_{22}\}$$

The constraints for the leader are given in Eq. (20) to Eq. (30).

$$g_1 = \frac{1}{x_1{x_2}^2x_3} - \frac{1}{27} \leq 0 \quad (20)$$

$$g_2 = \frac{1}{x_1{x_2}^2{x_3}^2} - \frac{1}{397.5} \leq 0 \quad (21)$$

$$g_3 = \frac{{x_4}^3}{x_2x_3{x_6}^4} - \frac{1}{1.93} \leq 0 \quad (22)$$

$$g_4 = \frac{{x_5}^3}{x_2x_3{x_7}^4} - \frac{1}{1.93} \leq 0 \quad (23)$$

$$g_5 = x_2x_3 - 40 \leq 0 \quad (24)$$

$$g_6 = \frac{x_1}{x_2} - 12 \leq 0 \quad (25)$$

$$g_7 = 5 - \frac{x_1}{x_2} \leq 0 \quad (26)$$

$$g_8 = 1.9 - x_4 + 1.5x_6 \leq 0 \quad (27)$$

$$g_9 = 1.9 - x_5 + 1.1x_7 \leq 0 \quad (28)$$

$$g_{10} = f_{12} - 1800 \leq 0 \quad (29)$$

$$g_{11} = f_{22} - 1100 \leq 0 \quad (30)$$

Each subsystem follower controls specific variables and aims to minimize the volume and stress of the corresponding gear shaft. Objective functions and constraints for subsystem1 are given in Eq. (31) to Eq. (38).

$$f_{1,1} = 0.7854x_1{x_2}^2\left(\frac{10{x_3}^2}{3} + 14.933x_3 - 43.0934\right) - 1.508x_1{x_6}^2 + 7.477{x_6}^3 + t_2 \quad (31)$$

$$f_{1,2} = \frac{\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 1.69 \times 10^7}}{0.1{x_6}^3} \quad (32)$$

Subject to:

$$g_{1,1} = g_3 = \frac{{x_4}^3}{x_2 x_3 {x_6}^4} - \frac{1}{1.93} \leq 0 \quad (32)$$

$$g_{1,3} = g_{10} = f_{12} - 1800 \leq 0 \quad (33)$$

$$\|y_1 - t_1\| \leq \varepsilon_1 \quad (34)$$

Where

$$y_1 = 0.7854 x_4 {x_6}^2 \quad (35)$$

$$X_1 = [x_4, x_6] \quad (36)$$

$$7.3 \leq x_4 \leq 8.3 \text{ , } 2.9 \leq x_6 \leq 3.9 \quad (37)$$

$$\varepsilon_1 = 10^{-3} t_1 \quad (38)$$

Objective functions and constraints for subsystem2 are given in Eq. (39) to Eq. (38).

$$f_{2,1} = -1.508 x_1 {x_7}^2 + 7.477 {x_7}^2 + t_1 \quad (39)$$

$$f_{22} = \frac{\sqrt{(\frac{745 x_5}{x_2 x_3})^2 + \sqrt{1.575 \times 10^8}}}{0.1 {x_7}^3} \quad (40)$$

Subject to:

$$g_{2,1} = g_4 = \frac{{x_5}^3}{x_2 x_3 {x_7}^4} - \frac{1}{1.93} \leq 0 \quad (41)$$

$$g_{2,2} = g_9 = 1.9 - x_5 + 1.1 x_7 \leq 0 \quad (42)$$

$$g_{2,3} = g_{11} = f_{22} - 1100 \leq 0 \quad (43)$$

$$\|y_2 - t_2\| \leq \varepsilon_2 \quad (44)$$

Where

$$y_2 = 0.7854 x_5 {x_7}^2 \quad (45)$$

$$\varepsilon_2 = 10^{-3} t_2 \quad (46)$$

The optimal solutions obtained from the followers are utilized to generate a comprehensive dataset. A neural network is trained to predict rational reaction sets (RRS) of the followers in response to the leader's decisions. This involves preprocessing the data, neural network architecture, training, and validation phases. The RRS obtained from the neural network are incorporated into the leader's optimization problem, allowing it to anticipate and account for the followers' responses in its decision-making process. The objective function values for $f_{1,1}, f_{1,2}, f_{2,1}, f_{22}$ are 2920,980,870,3021 respectively. These results are compatible with the reported ranges in [20].

IV. CONCLUSION

In conclusion, the study presents a novel numerical method for solving game theory problems in engineering design, leveraging AI-enhanced techniques to achieve efficient and robust optimization solutions. By leveraging AI-enhanced techniques, it has been demonstrated significant improvements in computational speed and efficiency compared to traditional methods. Future work will focus on extending the application of this method to address more complex engineering problems across diverse domains. This entails scaling up the analysis to encompass larger systems with a greater number of variables, objectives, and constraints, while also exploring the integration of advanced AI techniques such as deep learning and reinforcement learning. Additionally, investigating the impact of different problem formulations and optimization algorithms on solution quality and convergence speed will be crucial for advancing the understanding and applicability of this approach in practical settings.

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