

# Realistic Simulation of Error in Quantum Computing Circuits

Kevin M. Obenland  
MIT Lincoln Laboratory  
244 Wood Street, Lexington MA 02420  
kevin.obenland@ll.mit.edu

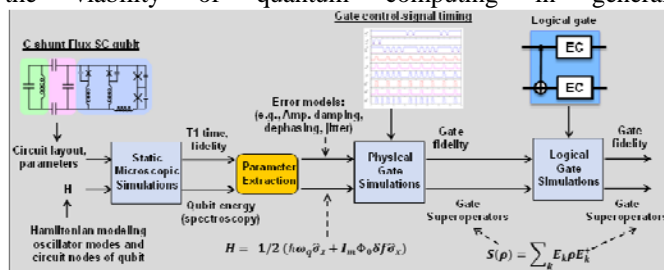
Andrew J. Kerman  
MIT Lincoln Laboratory  
244 Wood Street, Lexington MA 02420

**Abstract**—Realistic modeling of noise in quantum circuits is critical to the assessment of quantum logic gates and to the viability of quantum computing as a whole. In this work we develop realistic models of noise and use them in simulations of circuits with many physical qubits. This requires the development of new efficient modeling techniques and the use of parallel simulation methods.

**Keywords**—Monte Carlo simulation, quantum gates, quantum noise

## I. TECHNICAL SUMMARY

Just as the transistor is the fundamental building block of today's computers, the physical qubit is the base device out of which any future quantum computer will be constructed. Because of this, modeling the performance of these devices will be critical for understanding the viability of quantum computing. One of the most important aspects of performance for a qubit is its fidelity, or conversely the error rate of a logic gate. It is widely believed that achieving very low error rates for quantum devices, i.e., those close to that of the modern transistor will be very difficult. Therefore, fault-tolerant error-corrected gates will be required for any medium to large-scale quantum computer, and a realistic characterization of error at the physical device level is the first step in understanding the error of these error-corrected (or logical) gates and ultimately the viability of quantum computing in general.



**Figure 1: Simulation approach for modeling realistic noise processes in physical and logical gates based on superconducting qubits. A full-physics model based on the qubit Hamiltonian is used to extract a phenomenological model. This model is used to simulate dynamic physical gate operations, which are then characterized by gate superoperators.**

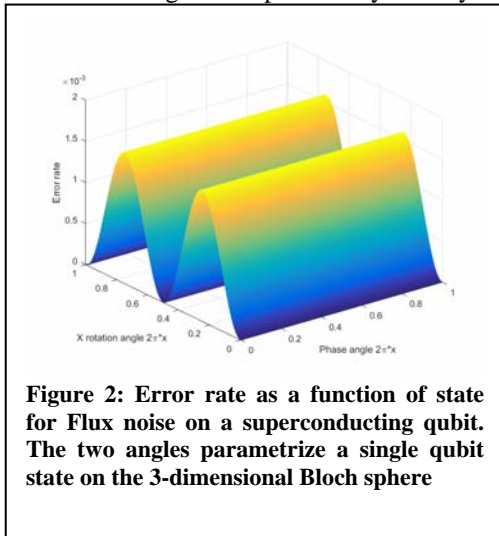
Our approach to realistic error modeling is to include the full physics of the qubit devices, however, to enable the modeling of larger systems we extract the salient details of the device physics to produce a phenomenological model that can be used to model dynamic gate operations subject to noise. An illustration of the different models and parameters of our technique is shown in Figure 1. To-date, approaches for modeling error in physical and logical qubits use idealized and average gate-error models. At the physical level, error is typically characterized using coherence times and the average error over a range of gates [1], and at the logical level classical stochastic error models are used because they allow for efficient simulations [2]. Neither of these models captures the correct quantum mechanical dynamics of the gates and error, and because of this, predictions using them may not be realistic. Our model of realistic noise is constructed as a function of the strength of different noise processes, spurious couplings, or control errors; some examples include: excited-state decay,  $1/f$  noise, timing jitter of control pulses, non-Markovian environments, and correlated classical control errors [3]. This level of simulation produces a full quantum process map (in the form of a superoperator that acts on an input density matrix) of each relevant few-qubit physical gate, as a function of the desired physical error parameters. The quantum process maps obtained from the physical simulation are then decomposed into a set of statistical operators (known as a Kraus operator decomposition.) This operator decomposition provides a more accurate description of the error than the stochastic models typically used in simulations of error-corrected gates. One example of our modeling approach is shown in Figure 2. Here we show the results of a simulation that incorporates flux noise with a  $1/f$  spectrum. The plot shows the error rate of a memory gate (100 ns) as a function of the input quantum state. As can be seen the error rate varies widely as a function of the state and the error level is only a function of the rotation between the  $|0\rangle$  and  $|1\rangle$  states and not the phase angle. In general, a gate with multiple simultaneous noise processes will exhibit error that is a function of both the rotation angle between qubit basis states and the phase angle. Another important factor is the dynamics of the gate that is being implemented, because error can be transformed by the operation of the gate. This can be captured by our modeling technique but is not properly modeled by

stochastic models, which model errors as operators applied between gates that are independent of the state.

## II. SUMMARY OF RESULTS

Our research consists of developing new efficient techniques to model the physics of the quantum computing devices of interest as well as to develop new computational simulation techniques. The work involves the following main topics:

- Techniques used to develop static models of superconducting qubit circuits. How the device components map to circuit nodes and how these are used to define a Hamiltonian description of the device.
- Description of the major noise processes present in the device and how these are modeled as phenomenological noise in the simulation.
- Numerical methods used in the simulation. The main numerical technique used in the simulation is the numerical integration of the Schrodinger equation.
- Parallel processing requirements and the computational techniques used. The complexity of the simulation grows exponentially with system size.



**Figure 2: Error rate as a function of state for Flux noise on a superconducting qubit. The two angles parametrize a single qubit state on the 3-dimensional Bloch sphere**

Additionally quantum Monte Carlo is used to model the noise processes. Both of these factors mean that efficient and parallel computational techniques must be developed to enable simulations of gates even with only 1-3 qubits.

- Error modeling and results. Comparison of the different types of noise and how this noise impacts the fidelity of the different gates implemented.
- Comparison of the physically realistic noise models to simplified classical noise models. Demonstration of the differences between the models and how this impacts the predictions when the noise operators are used in larger circuits.

## REFERENCES

- [1] E. Knill, D. Leibfried, R. Reichle, J. Britton, R. B. Blakestad, J. D. Jost, C. Langer, R. Ozeri, S. Seidelin, and D. J. Wineland, "Randomized benchmarking of quantum gates," *Phys. Rev. A* **77**, 012307 (2008)
- [2] D. Gottesman. *Stabilizer Codes and Quantum Error Correction*. Ph.D. thesis, California Institute of Technology, Pasadena, CA, (1997).
- [3] A.J. Kerman and W.D. Oliver, "High-Fidelity Quantum Operations on Superconducting Qubits in the Presence of Noise," *Phys. Rev. Lett.* **101**, 070501 (2008).